Noisy Label Detection and Counterfactual Correction

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***Abstract*—Data quality is of paramount importance to the training of any machine learning model. Recently proposed approaches for noisy learning focus on detecting noisy labeled data instances by using a fixed loss value threshold and excluding detected noisy data instances in subsequent training steps. However, a predefined, fixed loss value threshold may not be optimal for detecting noisy labeled data, whereas excluding the detected noisy data instances can reduce the size of the training set to such an extend that accuracy can be negatively affected. In this article, we propose Noisy label Detection and Counterfactual Correction (NDCC), a new approach that automatically selects a loss value threshold to identify noisy labeled data instances, and uses counterfactual learning to correct the noisy labels. To the best of our knowledge, NDCC is the first work to explore the use of counterfactual learning in the noisy learning domain. We demonstrate the performance of NDCC on several datasets under a variety of label noise environments. Experimental results show the superiority of the proposed approach compared to the state–of–the–art, especially in the presence of severe label noise.**

***Impact Statement*—The accuracy of machine learning models depends on training data quality. Quite unsurprisingly then, it drops dramatically (up to 53% in our experiments) as the percentage of noisy labels increases. The approach presented here is shown to maintain high performance even in the presence of highly corrupted data (i.e., 80% noisy labels) by performing joint noisy label detection and correction. Specifically, the proposed approach increases the accuracy rate of noisy label detection (up to 25%), while achieving a high noisy label correction rate (up to 72%). When presented with severe label noise (i.e., 80% noisy labels), the proposed approach lowers the noise rate to 52.5%. Beyond improving the accuracy of machine learning models that are trained with noisy label data, this research highlights the need to treat (as opposed to discard) noisy label instances during the training process.**

***Index Terms*—data quality, noisy learning, deep learning**

I. Introduction

MACHINE learning models have been applied in a wide range of applications, including, but not limited to, traffic prediction [1], face recognition [2], product recommendation [3] and online fraud detection [4]. Deep neural networks, one of the most popular branches of machine learning, have achieved remarkable performance to a variety of tasks due in part, to large quantities of human–annotated data [5], [6]. However, the label annotation process is labor–intensive, and often introduces label noise for reasons including insufficient information for low quality data, subjectivity in the labeling process, and limited number of expert annotators due to budgetary constraints [7]. After the completion of the data labeling process, identifying and correcting wrong labels is resource– and time–consuming. Furthermore, over–parameterized machine learning models, such as Deep Neural Networks, can overfit on noisy data instances by memorizing them during training [8], [9]. Learning and assessing machine learning models using noisy labels can result in biases and misleading accuracy reporting, with potentially detrimental results, such as wrong disaster diagnosis [10] or perpetuating biases in resource allocation (e.g., loan application) [11]. There are two common types of noise, namely: feature noise and label noise [12]. In this work, we focus on label noise which has been shown to be more harmful than feature noise [13].

To facilitate training a learning model over a noisy dataset, one commonly adopted approach is noise sample selection [14], which distinguishes the noisy from clean data instances during the training process, then excludes noisy instances from the training process [15]–[17]. In line with prior art, this work leverages loss to distinguish between noisy from clean data instances [18], [19]. The challenge is how to quantify the loss value during the training process. [20] ranks the loss value for all data instances and pre–sets the loss threshold with a specific noise rate (NR) to identify noisy data instances as those whose loss value is lower than the threshold. The key problem with that approach is twofold: (i) in the real–world, the noise rate is hard to estimate, and (ii) different choices of loss functions have different impacts on the loss value ranking. To address these challenges, we leverage peer loss [21] for noisy label detection. Specifically, peer loss is computed by substituting the current label with other possible labels in the label set, and does not require knowledge of the noise rate. Furthermore, since the comparison is among the same data instance with different labels, different loss functions do not affect the comparison result. [21] sets peer loss threshold to 0 to distinguish the noisy from clean data instances. However, our experiments (see Figure 4) show that 0 may not always be the optimal peer loss threshold. This article proposes an automated threshold selection approach to overcome this issue.

Upon detecting suspected noisy labeled data instances, these instances are typically excluded from the training process [22]. However, for small or severely noisy labeled datasets, excluding noisy data can dramatically reduce the size of the training set, to the point it becomes useless for training purposes. Furthermore, despite having noisy labels, the feature

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values of noisy labeled data instances are clean and could still be useful for training. This work is the first to explore the feasibility of correcting noisy labeled data instances by finding the true label using counterfactual learning. Specifically, for each detected noisy labeled instance, counterfactual data instances are computed for all possible labels. The label that achieves the minimum value of counterfactual score is then selected as the true label.

This work focuses on *training a robust learning model in the presence of noisy labeled data in the training set, through detecting and correcting noisy labeled data instances.* A new approach is proposed to (i) identify potentially noisy labeled data instances during training, (ii) estimate the true label of each detected noisy labeled data instance through counterfactual data generation, and (iii) output a robust learning model and revised dataset (i.e., with corrected labels). We evaluate the ability of the proposed approach to handle varying degrees of noisy labeled data using two benchmark datasets. In summary, the main contributions of this article are:

- Proposing a novel approach for automating the selection of the noisy peer loss threshold used to identify noisy labeled data during training.
- Introducing a practical approach for estimating the most probable true label for each detected noisy data instance using counterfactual learning.
- Demonstrating the superiority of the proposed solution against baselines using benchmark datasets under different noisy environments.

To ensure the reproducibility of our work, our method available at https://github.com/IDIASLab/NDCC.

II. Related Work

With the increase of complexity and scale of datasets, the possibility of including unreliable labels or noisy labels also increases. Training machine learning models with noisy labels significantly impacts their prediction performance. For this reason, a large variety of deep learning models for robust learning in noisy data environments has already been developed [23], [24]. For instance, the loss function–based approach in [23] minimizes the risk of unseen clean data with the presence of noisy labels in the training data. However, such loss function–based approaches are restricted to a particular framework, and thus, lack adaptability. Some approaches (e.g., [16], [21], [22], [24]) focus on selecting the true labeled instances from a noisy labeled dataset to mitigate the negative influence of noisy data instances. For instance, [21] uses peer loss to select clean data instances by fixing the loss threshold to 0. However, the optimal loss threshold may not always be fixed or predetermined. Instead of using a fixed threshold, this work learns the loss threshold for noisy labeled data instances detection during the training process itself.

After detecting suspected noisy labeled data instances, many approaches (e.g., [16], [22], [24]) exclude such instances in subsequent training steps. However, dropping suspected noisy label data instances can result in a diminished training set, while not taking advantage of the clean (and potentially useful) features of noisy labeled samples. [17] assigns more weight on clean than suspected noisy data instances. At the same time, mistreating noisy data instances as clean can lead to a highly inaccurate model. A label correction method is proposed in [25] but it considers the noise transition matrix to be known a priori for a given dataset. However, our proposed approach does not require prior knowledge of the noise transition matrix. We instead propose a counterfactual-based approach to correct the labels of suspected noisy labeled data instances. Counterfactual learning has been widely explored in explainable machine learning to shed light into how/why the output of a machine learning model would change if the input (i.e., features) were to change [25], [26]. Specifically, [27] leverages counterfactual learning to produce example–based explanations by feature perturbation. Feature perturbation may lead to different prediction results given a learning model; data instances with perturbed feature values (in our case labels) are considered counterfactual [28]. To the best of our knowledge, this work is the first to incorporate counterfactual learning directly into noisy learning.

Finally, training supervised machine learning models with crowdsourced data is inherently related to learning from noisy labels, and has been applied in a wide range of domains (e.g., [29]–[34]). For instance, [29] introduced an algorithm for learning a classifier, the accuracy of each annotator, and the actual true label of each data instance based on maximum likelihood estimation. More recently, [33] proposed multiple noisy label distribution propagation to tackle the problem of label integration from multiple sources, while accounting for intercorrelation between multiple label sets of various data instances. Last but not least, [34] proposed label noise robust support vector machine inference to achieve good model performance while reducing labeling costs. Different from the problem studied in this article, each data instance in crowdsource learning is associated with multiple labels, some of which may be noisy, since obtaining reliable labels can be time–consuming and/or costly. Instead, in our setting, each data instance is restricted to having only one label. This makes our task more challenging since no other label information can be referenced when the label of a given data instance is suspected to be noisy.

III. Preliminaries and Problem Statement

A. Notation

Let $D = (X, Y)$ denote a clean training dataset and $\tilde{D} = (X, \tilde{Y})$[1] a noisy dataset. N is the total number of data instances in D and $\tilde{D}$ (i.e., $X = \{\mathbf{x}_i\}_{i=1}^{N}$), and $\mathbf{x}_i \in X$ is an M dimensional feature vector. The total number of classes in both Y and $\tilde{Y}$ are K, and j denotes the class index. The label of $\mathbf{x}_i$ is denoted as $\mathbf{y}_i \in \mathbb{B}^K$, with entry j being set to 1 if $\mathbf{x}_i$ belongs to the jth class, and 0 otherwise. For example, for $K = 5$, $\mathbf{y}_i = [0, 1, 0, 0, 0]$ indicates that $\mathbf{x}_i$ belongs to Class 2. The task is to train a model f using $\tilde{D}$, since the clean dataset D is unavailable, to predict the true label y of previously unseen data instances. Let $\bar{y}$ denote the predicted outcome. To minimize the influence of noisy data on the model

[1]Data instances in $\tilde{D}$ are either clean or noisy labeled. Same with $\tilde{Y}$.

TABLE I
EXPLANATION OF MAIN SYMBOLS USED IN THIS ARTICLE.

Symbol	Description
N	Total number of data instances. (A data instance is denoted by index i)
M	Total number of features for each data instance. (A feature is denoted by index m)
K	Total number of classes
j	jth class
$\tilde{D}$	Noisy dataset
D_{pre}	Clean pretrained dataset in Algorithms 1 and 3
$\hat{D}$	Revised dataset
X_n/X_c	Detected noisy/clean dataset in Algorithm 3
h_c/h_n	Objective function for noise detection/correction
g	Pre–trained model obtained by training with D_{pre} in Algorithms 1 and 3
$f(\mathbf{W})$	Learning model with weight matrix $\mathbf{W} \in \mathbb{R}^{K\times M}$
l	Categorical cross entropy loss
$\hat{\mathbf{x}}_i^j$	Counterfactual data of $\mathbf{x}_i$ with target label j
$\hat{\mathbf{y}}_i^j$	Counterfactual label when considering target label j for $\mathbf{x}_i$
$\tilde{\mathbf{y}}_i$	Noisy label vector of $\mathbf{x}_i$
$\tilde{\mathbf{y}}_i^j$	Noisy label of $\mathbf{x}_i$ with label j
ϕ_i	Indicator of data instance i being clean or noisy
T_{pre}	Training epoch for pre–train model g in Algorithms 1 and 3
T_{cf}	Counterfactual search epoch in Algorithms 2 and 3
T_n	Training epoch in Algorithm 3 step 25
T_{all}	Training epoch for NDCC in Algorithm 3

performance, we propose an approach to detect noisy data instances, and assign them with the most likely true label while learning f. We leverage counterfactual learning to search for the most likely true label for each noisy data instance. Specifically, each noisy data instance is associated with K counterfactual data instances $(\hat{\mathbf{x}}_i^j, \hat{\mathbf{y}}_i^j)$, each generated for each label $\hat{\mathbf{y}}_i^j$, where $j \in 1, 2, 3, .., K$. By comparing $(\mathbf{x}_i, \tilde{\mathbf{y}}_i)$ with each $(\hat{\mathbf{x}}_i^j, \hat{\mathbf{y}}_i^j)$, we determine the most likely true label $\hat{\mathbf{y}}_i^j$, and substitute the noisy label with $\hat{\mathbf{y}}_i^j$ (see Section IV-C). Table I summarizes the notation used hereafter.

B. Problem Statement

The goal of this work is to learn a robust classifier f over a noisy labeled dataset by minimizing the influence of noisy labeled data instances during training. To achieve this goal, we split the problem into three sub–problems: (i) learn a classifier f that accurately maps X to Y, (ii) detect noisy labeled data instances, and (iii) assign the most likely true label to suspected noisy labeled data instances through counterfactual learning. For clarity, *clean* data instances refer to data instances with correct labels; *noisy* data instances refer to data instances with wrong labels; and observed labels can be either clean or noisy.

IV. PROPOSED APPROACH

We propose Noisy label Detection and Counterfactual Correction (NDCC), a novel approach for training a robust classifier over a noisy labeled dataset. The objective function of NDCC follows:

$$\arg\min_{\mathbf{W},\hat{\mathbf{x}}_i^j} \sum_{i=1}^{N} \phi_i h_c(\mathbf{W}, \mathbf{x}_i) + (1-\phi_i) h_n(\mathbf{W}, \hat{\mathbf{x}}_i^j), \qquad (1)$$

where ϕ_i is the noisy labeled data instance indicator, $h_c()$ denotes the function for detecting noisy labeled data instances, and $h_n()$ refers to the noisy label correction functions. Both h_n and h_c are discussed in detail in Sections IV-A and IV-C, respectively. The value of ϕ_i can be either 1 or 0. Specifically, if x_i is considered to be a clean data instance, then ϕ_i is set to 1. As a result, only h_c needs to be computed for clean data instances. On the other hand, if x_i is considered to be a noisy instance, ϕ_i is set to 0, meaning that for each noisy data instance, only h_n needs to be computed.

Overall, Eq. (1) can be viewed as a combinatorial optimization problem, which is difficult to solve directly. We therefore solve Eq. (1) by alternatively searching for the optimal solutions of $\mathbf{W}$ and $\hat{\mathbf{x}}_i^j$. Convexity is proven in Appendix A. In each round, we accomplish the alternative search in two steps, as follows: (i) noisy label calibration (i.e., search solutions for $\hat{\mathbf{x}}_i^j$), which requires noisy label detection (Section IV-A) and label correction by counterfactual generation (Section IV-C), and (ii) model training (i.e., search solutions for $\mathbf{W}$).

Figure 1 provides an overview of NDCC. Initially, the noisy dataset $\tilde{D} = (X, \tilde{Y})$ is provided as input to the noisy label detection module, which then outputs suspected noisy label data instances $(X_n, \tilde{Y})$, and sets the noisy indicator $\phi = 0$ for each $\mathbf{x}_i \in X_n$, and 1 for each $x_i \in X_c$. Therefore, $\phi_i h_c(\mathbf{W})$ in Eq. (1) reflects the loss for clean data instances, whereas $(1 - \phi_i)h_n(\mathbf{W}, \hat{\mathbf{x}}_\mathbf{i})$ reflects the loss for noisy labeled data instances. The label counterfactual correction module assigns each $\mathbf{x}_i \in X_n$ with the most likely true label $\hat{\mathbf{y}}_i$, then substitutes $\tilde{D}$ with the label–revised dataset $\hat{D}$, to be used in subsequent rounds of training $f(W)$. Note that $\hat{D}$ can be updated multiple times through the training process, as additionally noisy labeled data instances are identified.

A. Noisy Label Detection (h_c)

Loss can identify noisy labeled data instances [15], [19], [35]. Specifically, [15] pointed out that the loss of clean data instances is expected to be lower than that of noisy labeled data instances, mainly because noisy labeled data instances are often outliers with respect to the distribution of clean data, and the learning model tends to make predictions different from the noisy labels. Experimental results presented in Figure 2 support this claim by showing that the loss value for a large number of noisy labeled data instances is higher than that of clean data instances, even under different label noise environments (i.e., symmetric[2] and asymmetric noise[3]).

The question then is how to determine a loss threshold to distinguish between clean and noisy labeled data instances. A relative "large" or "small" loss can be manually specified by inspecting the overall loss value distribution. However, a classifier cannot automatically determine whether the loss score is "large" or "small" without knowing the overall loss value distribution. Furthermore, having a pre–set and fixed loss threshold is impractical, as the loss distribution may vary across different classification tasks. Additionally, the correct

[2]The true label flips to all other labels with equal probability.

[3]A noisy label is generated by flipping the true label j to class $j + 1$ [21].

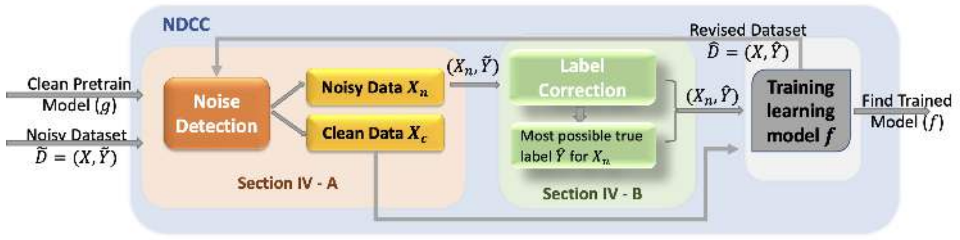
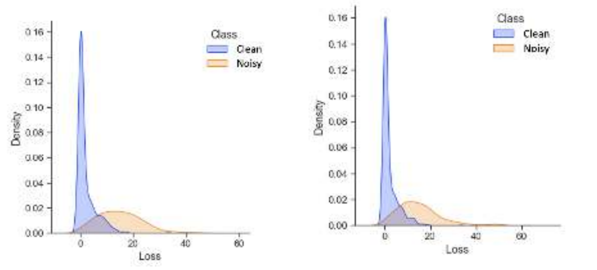

NDCC
Revised Dataset $\hat{D} = (X, \hat{Y})$
Clean Pretrain Model (g)
Noisy Dataset $\tilde{D} = (X, \tilde{Y})$
Noise Detection
Noisy Data X_n
Clean Data X_c
$(X_n, \tilde{Y})$
Label Correction
Most possible true label $\hat{Y}$ for X_n
$(X_n, \hat{Y})$
Training learning model f
Find Trained Model (f)
Section IV - A
Section IV - B

Fig. 1. Visualization of NDCC.



(a) Symmetric noise, NR = 0.4 (b) Asymmetric noise, NR = 0.4

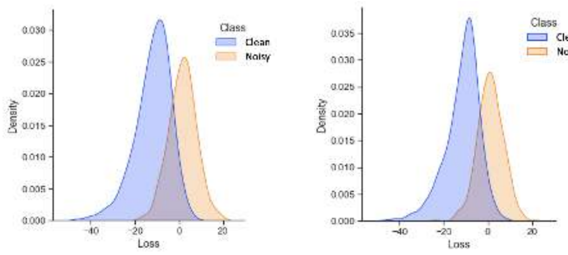
Fig. 2. Loss value distribution for CIFAR–10 with respect to different noisy environments. $x-$axis represents the loss score, and $y-$axis represents the frequency of a particular loss score in the $x-$axis.

labels of noisy data instances are usually unavailable, making it impossible to pre–select a suitable loss threshold.

Of particular relevance to this problem, [36] proposed peer loss, defined as $L_{PL} = l(f(\mathbf{W}, \mathbf{x}_i), \mathbf{y}_i) - l(f(\mathbf{W}, \mathbf{x}_i), \mathbf{y}_i^j)$, where $l(f(\mathbf{W}, \mathbf{x}_i), \mathbf{y}_i)$ is the loss with respect to given label $\mathbf{y}_i$, and $l(f(\mathbf{W}, \mathbf{x_i}), \mathbf{y}_i^j)$ is the loss with respect to a possible random label $\mathbf{y}_i^j$, different from $\mathbf{y}_i$. Based on the peer loss, [21] defined the loss value threshold, which takes all possible label values into consideration in order to identify data instances with high loss as noisy labeled. Inspired by this idea, we define the objective function for detecting noisy labeled data instances as [21]:

$$h_c(\mathbf{W}, \mathbf{x}_i) = l(f(\mathbf{W}, \mathbf{x_i}), \tilde{\mathbf{y}}_i) - \frac{1}{K}\sum_{j=1}^{K}(l(f(\mathbf{W}, \mathbf{x}_i), \mathbf{y}_i^j), \quad (2)$$

where f is the learning model with parameters $\mathbf{W}$, $l(f(\mathbf{W}, \mathbf{x}_i), \tilde{\mathbf{y}}_i)$ denotes the loss value of the observed label, and $\frac{1}{K}\sum_{j=1}^{K}(l(f(\mathbf{W}, \mathbf{x}_i), \mathbf{y}_i^j)$ is the average loss value of all possible K labels. Figure 3 confirms that the peer loss value for the majority of the clean data instances is smaller than that of noisy labeled data instances. Nevertheless, we consider a hypothetical scenario, in which the loss of a clean data instance is very high. In this scenario, the loss for $l(f(\mathbf{W}, \mathbf{x}_i), \mathbf{y}_i^j))$ is high. If the loss value for this data instance with other possible labels is also high, indicating that the model f is uncertain about $\mathbf{x}_i$, then the average loss value, $\frac{1}{K}\sum j = 1^K(l(f(\mathbf{W}, \mathbf{x}_i), \mathbf{y}_i^j))$, is also high. Despite the large loss, the final value $h_c(\mathbf{W}, \mathbf{x}_i)$ is likely to be below or close to the threshold, which separates clean and noisy labeled data instances. That is because the first term and the second term in h_c are both high, therefore, the final value of h_c should be small. If the loss value for this data instance with other possible labels is low, then $\frac{1}{K}\sum_{j=1}^{K}(l(f(\mathbf{W}, \mathbf{x}_i), \mathbf{y}_i^j)$ is lower than $l(f(\mathbf{W}, \mathbf{x}_i), \mathbf{y}_i^j))$, meaning that the learning model f considers $\tilde{\mathbf{y}}_i$ as the least likely true label, even though it is the true label. In this case, the model considers the data instance as noisy ($\phi_i = 0$) based on Eq. 2, and tries to find its possible true label, resulting in a loss value based on $h_n(\mathbf{W}, \hat{\mathbf{x}}i^j)$ according to Eq. 1. If the true label is found correctly, this scenario does not affect the overall learning process due to the low value of h_n. However, if the true label cannot be found correctly, the overall learning process will be impacted by wrongly updating the label of a clean data instance due to the high value of h_n. Our experiments confirm that even if labels of clean data instances are wrongly updated (often during the initial phase of training, when f is not particularly accurate), NDCC learns to correct many such mistakes in subsequent rounds, as it learns a more accurate classification model.



(a) Symmetric noise, NR = 0.4 (b) Asymmetric noise, NR = 0.4

Fig. 3. Peer loss value (i.e., computed by Eq. (2)) distribution for CIFAR–10 with respect to different noisy environments. $x-$axis corresponds to peer loss score, and $y-$axis corresponds to frequency with respect to particular peer loss score in $x-$axis.

B. Noisy Label Threshold Selection Criterion

After computing h_c, the question is how to use it to detect noisy labeled data instances. [21] considers data instances with $h_c \geq 0$ to be noisy labeled. This is because the loss of the observed label $\tilde{y}_i$ is larger than the average loss of the other possible labels y_i^j [21]. However, 0 need not be the optimal loss value threshold. For instance, Figure 4 shows the peer loss of $1,000$ randomly selected CIFAR–10 data instances, under symmetric noise (NR = 0.1). The red dot line (peer loss threshold of 0) is evidently not optimal compared to the black dot line, which can detect more noisy labeled data instances.

This work proposes to automate the peer loss threshold selection by considering as noisy labeled those data instances

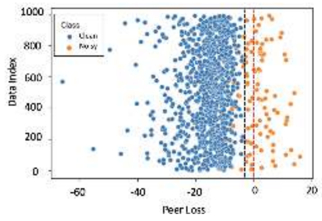


Fig. 4. Peer loss value distribution for random selected 1,000 data instances in CIFAR–10 under symmetric noise (NR = 0.1). $x-$axis corresponds to peer loss score, and $y-$axis corresponds to each data instance.

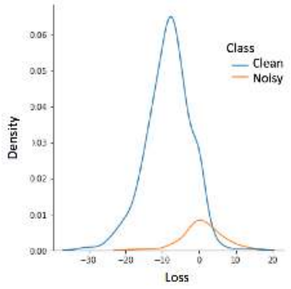


Fig. 5. Peer loss value distribution without pre–trained model with random selected 1,000 data instances in CIFAR–10 under symmetric noise (NR = 0.1). $x-$axis corresponds to peer loss score, and $y-$axis corresponds to frequency with respect to particular peer loss score in $x-$axis.

whose loss is large, but not exactly larger, than their average label loss threshold, as shown in Figure 4.

Before elaborating on our proposed approach, we note that using a randomly initialized deep neural network as a starting point can lead to erroneous loss estimation. For instance, Figure 5 shows that the loss of clean and noisy data instances may overlap. Erroneous loss estimation can lead to missdetection of clean instances as noisy (and visa versa), introducing even more noisy labeled data instances into the training dataset. Therefore, the starting point of a classification model is crucial. Inspired by [37], which showed that a small portion of clean labels improves the model robustness in noisy detection, we pre–train a model g (see Section VI-A3 for a detailed discussion on g), using a small portion of data instances, denoted as D_{pre}, whose labels are guaranteed to be accurate. In the real–world, clean data instances can be obtained from experts [38].

To automatically detect and revise noisy data instances in the overall training set, we begin by calculating h_c for each data instance in D_{pre} using g (we denote this as l_{pc}). Next, since the type of label noise in the training dataset may be unknown, we randomly select 10% of the data instances in D_{pre}, and artificially introduce noise by randomly switching their label to a different one. The noisy version of the pre–train dataset is denoted as $\tilde{D}_{pre} = \tilde{D}^c_{pre} \cup \tilde{D}^n_{pre}$, where $\tilde{D}^c_{pre}$ ($\tilde{D}^n_{pre}$) is the set of clean (noisy) data in $\tilde{D}_{pre}$. Next, we calculate h_c on $\tilde{D}_{pre}$ and record the loss as l_{pn}. The difference between l_{pc} and l_{pn} (i.e., $l_{diff} = l_{pc} - l_{pn}$) is used to define the loss area $\tilde{D}_{ns} = \{\mathbf{x}_i | l_{diff}(\mathbf{x}_i) \leq \min_{\mathbf{x}_q \in \tilde{D}^c_{pre}} l_{diff}(\mathbf{x}_q), \forall \mathbf{x}_i \in \tilde{D}_{pre}\}$. The rationale for calculating $\tilde{D}_{ns}$ is that it may contain the majority of noisy data instances, since l_{diff} is smaller for noisy data instances compared with clean data instances because of higher l_{pn}. As illustrated by Figure 6, the absolute value of the loss difference l_{diff} for noisy data instances is higher than the clean data instances.

In subsequent steps in the training process (i.e., without using $\tilde{D}_{pre}$), we have no prior indication about which data instances are clean or noisy. Figure 7 shows that noisy data instances are more likely to reside in $\tilde{D}_{ns}$, in a real training experiment with $\tilde{D}$. This observation lets us estimate the peer loss threshold by calculating the average loss, as follows:

$$thr = \frac{1}{|\tilde{D}_{ns}|} \sum_i h_c(\mathbf{W}_g, \mathbf{x}_i), \mathbf{x}_i \in \tilde{D}_{ns}, \tag{3}$$

where $\mathbf{W}_g$ denotes the parameters of the clean pre–trained model g. Algorithm 1 is proposed to compute$\tilde{D}_{ns}$, which is used to calculate the noisy label threshold based on Equation 3. thr is initially set to 0, as per [21].

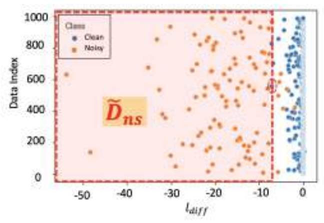
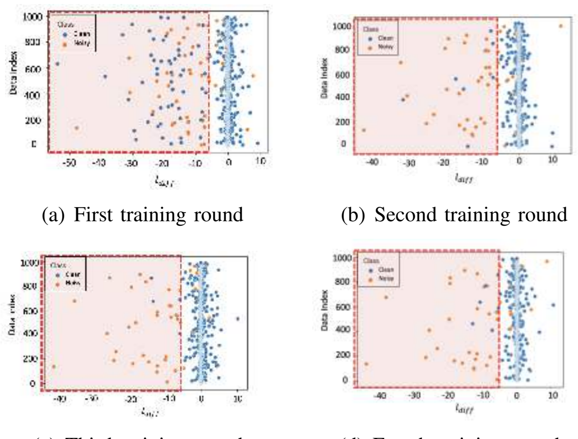


Fig. 6. Pre–train experiment with $\tilde{D}_{pre}$. $x-$axis corresponds to l_{diff}, and $y-$axis corresponds to each data instance. The red circled data instances define the upper bound of $\tilde{D}_{ns}$.



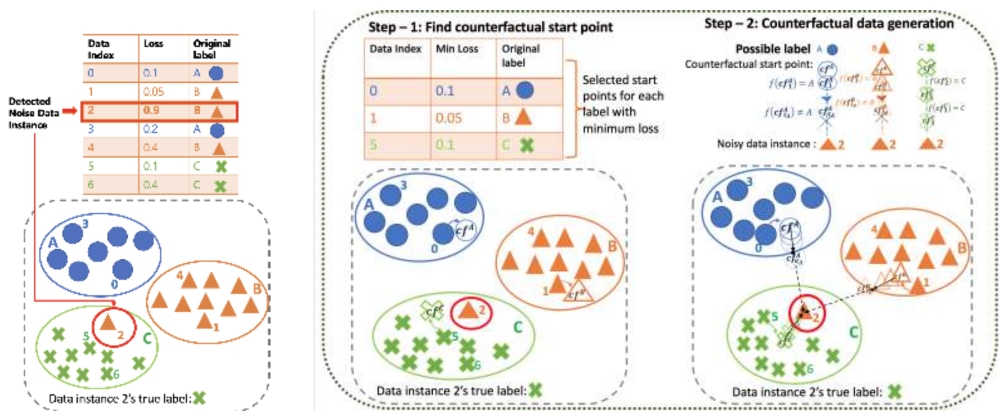
(a) First training round (b) Second training round

(c) Third training round (d) Fourth training round

Fig. 7. Test $\tilde{D}_{ns}$ on training simulation with $\tilde{D}$ in different learning rounds. $x-$axis corresponds to loss score, and $y-$axis corresponds to frequency with respect to particular loss score in $x-$axis. The training around corresponds to T in algorithm 3.

C. *Noisy Label Correction h_n*

The noisy label correction process is designed to pair the noisy labeled data instances with their most likely true label using counterfactual learning. Counterfactual learning is used to explain algorithmic decisions by feature perturbation [27], [28], [39]. This work generates a counterfactual data instance with other possible labels for each detected noisy labeled data instance $(\mathbf{x}_i, \tilde{\mathbf{y}}_i) \in X_n$. Specifically, the noisy label detection


Data Index
Loss
Original label
0 0.1 A
1 0.05 B
2 0.9 B
3 0.2 A
4 0.4 B
5 0.1 C
6 0.4 C
Detected Noise Data Instance
A
B
C
Data instance 2's true label:
Step – 1: Find counterfactual start point
Data Index
Min Loss
Original label
0 0.1 A
1 0.05 B
5 0.1 C
Selected start points for each label with minimum loss
Step – 2: Counterfactual data generation
Possible label
Counterfactual start point:
Noisy data instance :

(a) Example for loss value (b) Example for counterfactual label searching

Fig. 8. Illustration of counterfactual noisy label correction.

Algorithm 1 $\tilde{D}_{ns}$ Computation

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Input: Clean pre–trained model $g$, clean data subset $D_{pre}$,
    input noise rate $\tau_p$, learning epoch $T_{pre}$
1: Select the number of $|D_{pre}| \times \tau_p$ data instances from $D_{pre}$,
    and randomly permute their labels
2: Compute the loss value $l_{pc}$ using Eq. 2 and $g$
3: Train model $\tilde{g}$ (with $g$ as a starting point) for $T_p$ epochs,
    using data instances in $\tilde{D}_{pre}$
4: Compute the loss value $l_{pn}$ using Eq. 2 and $\tilde{g}$
5: Compute $l_{diff} = l_{pc} - l_{pn}$
6: Set the smallest value of $l_{diff}$ of the clean data instances
    as the loss value threshold for $\tilde{D}_{ns}$
Output: $\tilde{D}_{ns}$
```

Algorithm 2 Label Correction with Counterfactual Learning

```
Input: $(x_i, \tilde{y}_i)$, target label set $Y = \{1, 2, ..., j, ...K\}$,
    $T_{cf}$ maximum epoch number, learning model
    $f(\mathbf{W})$, and counterfactual starting point set
    $\{x^1_{cf_0}, x^2_{cf_0} ..., x^j_{cf_0}, ..., x^K_{cf_0}\}$
1: Initialize optimal counterfactual set $X_{cf} = \emptyset$
2: for each $j \in Y$ do
3:     Set $x^j_{cf_0}$ as the counterfactual starting point and $t = 1$
4:     while $(f(W, x^j_{cf_t}) = \mathbf{y}^j_i$ and $t \leq T_{cf})$ do
5:         Optimize the loss using $x^j_{cf_t}$ and $x_i$ based on Eq. (4)
6:         $t = t + 1$
7:     end while
8:     Select $\hat{\mathbf{x}}^j_i = x^t_{cf_j}$ that minimizes Eq. (4)
9:     Add $\hat{\mathbf{x}}^j_i$ to $X_{cf}$
10: end for
Output: Output $\hat{\mathbf{x}}^j_i$ in $X_{cf}$ which achieves the minimum
    value of $h_n$, and the corresponding value of $h_n(\hat{\mathbf{x}}^j_i)$
```

module in IV-A provides the loss for each data instance, as shown in Figure 8(a). In this example, data instance 2 has the highest loss of 0.9. Thus, $(\mathbf{x}_2, \tilde{\mathbf{y}}_2)$ is suspected to be noisy labeled. The true label $\mathbf{y}_2$ for $\mathbf{x}_2$ belongs to the label set $Y = \{A, B, C\}$. We consider noisy label B as a viable candidate because the noise detection module may make a wrong detection by treating clean data instances as noisy. Therefore, the target counterfactual data instances include $(\hat{\mathbf{x}}_2^{j=A}, \hat{\mathbf{y}}_2^{j=A})$, $(\hat{\mathbf{x}}_2^{j=B}, \hat{\mathbf{y}}_2^{j=B})$, $(\hat{\mathbf{x}}_2^{j=C}, \hat{\mathbf{y}}_2^{j=C})$.

The following question is how to generate the counterfactual data instances for a detected noisy labeled data instance. One commonly used counterfactual generation criterion is the **Proximity Score** [39] which evaluates the distance between the counterfactual data $\hat{\mathbf{x}}^j_i$ and the original feature vector $\mathbf{x}_i$. A smaller distance between the data instance $\mathbf{x}_i$ and its counterfactual data instance $\hat{\mathbf{x}}^j_i$ represents a higher probability that the true label of $\mathbf{x}_i$ is the target class[4] j for $\hat{\mathbf{x}}^j_i$. The proximity measure is computed as:

$$h_n = dist(\hat{\mathbf{x}}^j_i, \mathbf{x}_i), \tag{4}$$

where $dist()$ denotes Euclidean distance. This work first selects the data instance with the minimum loss value (i.e., highest confidence of correct classification) as the counterfactual starting point (i.e., $\hat{\mathbf{x}}^A_2$, $\hat{\mathbf{x}}^B_2$, and $\hat{\mathbf{x}}^C_2$) for each possible label, as illustrated in Figure 8(b) step–1. Next, we minimize the proximity score by perturbing the feature values of $\hat{\mathbf{x}}^j_i$ (i.e., $\hat{\mathbf{x}}^A_2$, $\hat{\mathbf{x}}^B_2$, and $\hat{\mathbf{x}}^C_2$) and forcing it to get closer to the target noisy data instance. However, without any limitation, $\hat{\mathbf{x}}^j_i$ will eventually become equal to $\mathbf{x}_i$, reaching a proximity score of 0. To tackle this issue, we use **Validity Score** [27] (a measure of the degree of validity of a counterfactual data instance) as a stopping criterion for the counterfactual data instance generation process. Specifically, we use 1 (i.e., highest value) as our stopping criterion to guarantee that the generated counterfactual data instance $\hat{\mathbf{x}}^j_i$ belongs to the particular class j. A higher validity score represents higher confidence (i.e., lower loss) of the predictor outputting the target label $\hat{\mathbf{y}}^j_i$ for the counterfactual data instance $\hat{\mathbf{x}}^j_i$, with $\hat{\mathbf{x}}^j_i$ being absolutely valid if the prediction outcome is the same as the target label (i.e., $f(\hat{\mathbf{x}}^j_i) = \hat{\mathbf{y}}^j_i$). Taking label A in Figure 8(b) as an example, after perturbing the feature of $\hat{\mathbf{x}}^A_2$ for t_A times, we obtain counterfactual data instance $\hat{\mathbf{x}}^A_{2_{t_A}}$, which triggers the stopping criterion because $f(\hat{\mathbf{x}}^A_{2_{t_A}}) \neq A$. The final output counterfactual data instance for label A is $\hat{x}^A_{2_{t_A - 1}}$

[4]In counterfactual learning, the generated data instance can be classified into a particular class (i.e., target class) by the learning model.

(i.e., $f(\hat{x}^A_{2_{t_A-1}}) = A$). The same process is carried out for labels B and C. After obtaining all valid counterfactual data instances, the most likely true label for the noisy data instance $\mathbf{x}_i$ is determined to be the label of the counterfactual data instance with the smallest proximity score. The overall noisy label correction process is described in Algorithm 2.

D. NDCC Algorithm

Algorithm 3 summarizes the process of learning a model from potentially noisy labeled data based on Eq. (1). Initially, a pre–trained model g is used to generate $\tilde{D}_{ns}$ for automatically selecting the loss threshold in each following learning round (step 2). Then, potentially noisy labeled data instances in $\tilde{D} = (X, \tilde{Y})$ are detected (steps $6-14$). The most likely true label for each noisy data instance is determined using counterfactual label correction, and the dataset is updated with the revised labels (steps $19-23$). The revised dataset (step 24) is used to train model f (step 25). The noisy loss threshold for the next iteration is determined using the trained model f (step $27-$ -29). The algorithm terminates when it reaches the maximum training epoch T_{all}, or if no updates to the dataset occur.

E. Interrelationship of Counterfactual Label Correction and Accuracy of Learned Model

Both Eq. (1) and Algorithm 3 suggest that the accuracy of the counterfactual label correction process (Section IV-C) depends on the accuracy of the current model (Section IV-D), and vice versa. Therefore, a highly inaccurate model f can lead to erroneous label "corrections" for clean data instances treated as noisy, and missed true noisy labeled data instances, which can in turn cause further degradation of f. We avoid this scenario by pre–training a model g (used as a starting point in Algorithm 3) using a subset of training data that are verified to be clean, as detailed in Section IV-B. Similarly, a model f with medium confidence can mislead the counterfactual label correction module into making mistakes (e.g., pairing detected noisy data instance with the wrong label). Thankfully, such mistakes do not increase the number of noisy labeled data instances, since the suspected data instances were initially noisy labeled. Our experiments suggest that with a reasonable true correction rate (up to 70% for CIFAR–10), the counterfactual label correction model eventually provides more clean data instances, leading in turn to an accurate learning model. In the best case scenario, i.e., when the learning model f is highly accurate, the counterfactual label correction module also achieves a high true correction rate, increasing the number of clean data instances in the training set, and in turn, potentially further improving the accuracy of f.

Algorithm 3 NDCC

```
Input: Noisy dataset $\tilde{D}$, pre–train clean dataset $\tilde{D}_{pre}$, pre–train clean model $g$, learning model $f$, pre–set learning epoch $T_{pre}$, maximum epoch for learning model $T_n$, maximum epoch for counterfactual searching $T_{cf}$, and learning round $T_{all}$
1:  $t \leftarrow 1$, $thr_1 \leftarrow 0$, $\mathbf{W_t}$, $\tilde{D}^t \leftarrow \tilde{D}$
2:  Compute the noisy loss region $\tilde{D}_{ns}$ using Algorithm 1
3:  Calculate $l_{pc_i}$ by Eq. 2 with $g$ for each $\mathbf{x}_i \in \tilde{D}^t)$
4:  while ($t \leq T_{all}$ and $\tilde{D}^t$ equals to $\tilde{D}^{t-1}$) do
5:      /*Noise Detection Section*/
6:      for $\mathbf{x}_i \in \tilde{D}^t$ do
7:          $X_c^t, X_n^t \leftarrow \emptyset$
8:          Calculate $h_c(\mathbf{x}_i, \mathbf{W}_t)$ using Equation (2)
9:          if $h_c(\mathbf{x}_i, \mathbf{W}_t) \leq thr_t$ then
10:             Set $\phi_i^t \leftarrow 1$, and add $\mathbf{x}_i$ into $X_c^t$
11:         else
12:             Set $\phi_i^t \leftarrow 0$, and add $\mathbf{x}_i$ into $X_n^t$
13:         end if
14:     end for
15:     if $X_n^t = X_n^{t-1}$ and $t >= 2$ then
16:         break
17:     end if
18:     /*Noise Correction Section*/
19:     Select data instances with minimum value of $h_n$ for each label and set these as counterfactual starting points $\{x^1_{cf_0}, x^2_{cf_0} ..., x^j_{cf_0}, ..., x^K_{cf_0}\}$
20:     for Each $\mathbf{x}_i \in X_n^t$ do
21:         Output $\hat{\mathbf{x}}_i^j$ and $h_n(\hat{\mathbf{x}}_i^j)$ using algorithm 2
22:         Update label of $\mathbf{x}_i$ with $j$
23:     end for
24:     Get label revised dataset as $\tilde{D}^{t+1}$
25:     Train the learning model $f(\mathbf{W}_t)$ with updated $\tilde{D}^t$ for $T_n$ epochs by minimizing the cross entropy loss and output $f(\mathbf{W}_{t+1})$
26:     /*Updating Loss Threshold*/
27:     Calculate $l^t_{pn_i}$ by Eq. 2 with $f(\mathbf{W}_{t+1})$ for $\mathbf{x}_i \in \tilde{D}_{pre}$
28:     Calculate $l^t_{diff_i} = l_{pc_i} - l^t_{pn_i}$ for $\mathbf{x}_i$ in $\tilde{D}_{pre}$
29:     Compute $thr_{t+1}$ by Eq. (3)
30:     $t \leftarrow t + 1$
31: end while
Output: $f(\mathbf{W}_T)$ and $\tilde{D}^{T_{all}}$
```

V. Complexity Analysis

A. Computation of $\tilde{D}_{ns}$: The noisy threshold selection process (Section IV-B) comprises three steps. First, the labels of $|D_{pre}| \times \tau_p$ randomly selected data instances are shuffled, and their loss value is computed (Algorithm 1 steps: 1–2) in $\mathcal{O}(N_{pre}\tau_p)$, where $N_{pre} < N$ denotes the number of the data instances in D_{pre}. Next, model g (i.e., ResNet34) is trained (Algorithm 1 step: 3) in $\mathcal{O}(N_{pre}whC_hk^2dc)$, where C_h is the number of channels, w and h are the width and height of the input data, k is the size of the filter, and c and d are the number and spatial dimension of filters, respectively. Finally, l_{diff} and $\tilde{D}_{ns}$ are computed in $\mathcal{O}(N_{pre} + 1)$. Thus, the overall complexity of $\tilde{D}_{ns}$ computation is $\mathcal{O}(N_{pre}whC_hk^2dc)$ (i.e., dominated by the training time of ResNet34).

B. Counterfactual Data Generation: Counterfactual data are generated for each data instance in N_{ns}, whose label is to be corrected, in $\mathcal{O}(wh)$ in each iteration $\leq T_{cf}$ (Algorithm 2 steps: 5–6). Therefore, the overall complexity for counterfactual data generation is $\mathcal{O}(N_{ns}KwhT_{cf})$.

C. Overall NDCC Complexity: Algorithm 3 comprises six steps. First, the noisy label threshold is computed (Algorithm 3 steps: 1–3) in $\mathcal{O}(N_{pre}whC_hk^2dc)$. Next, in each training iteration t, h_c is computed for all data instances in $\mathcal{O}(NK)$, where N denotes the total number of data instances in $\tilde{D}$ (Algorithm 3 steps: 6–17). Then, the starting points for counterfactual data generation are selected in $\mathcal{O}(N)$ (Algorithm 3 step: 19). After selecting the starting points, counterfactual labels are produced in $\mathcal{O}(N_{ns}whKT_{cf})$ (Algorithm 3 steps: 20–24). Next, ResNet34 is trained on the updated dataset in $\mathcal{O}(NwhC_hk^2dc)$. Finally, the noisy threshold is recalculated in $\mathcal{O}(N)$ (Algorithm 3 steps: 27–29). Thus, the overall complexity of NDCC is $\mathcal{O}(T_{all}NwhC_hk^2dc + T_{all}N_{ns}whKT_{cf})$.

VI. Evaluation

A. Setting

1) Datasets: We evaluate NDCC on three widely used benchmark datasets [40]–[42] and a real–world noisy dataset [43]. **CIFAR–10 [44]:** An image dataset pre–splitted into a training/test set of $50,000$ and $10,000$ instances, accordingly. Each data instance (a $32 \times 32 \times 3$ colorful image) is associated with 10 classes (i.e., airplane, automobile, bird, cat, deer, dog, frog, horse, ship, truck). **CIFAR–100 [44]:** An image dataset that is similar to CIFAR–10, and contains 100 classes of data, with each class having 600 instances. The 100 classes are grouped into 20 superclasses. Every image has a "fine" label representing its class as well as a "coarse" label representing its subclasses. **Fashion–MNIST [43]:** Real–world dataset of 28×28 grayscale images, each associated with one of 10 classes (i.e., t-shirt/top, trouser, pullover, dress, coat, sandal, shirt, sneaker, bag, ankle boot), pre–split into a training/test set of $60,000$ and $10,000$ data instances, respectively. **Clothing1M [40]:** Large–scale real–world clothing recognition dataset comprising over 1 million images of clothing items with noisy labels.

2) Noise Environment: With the exception of Clothing1M, all datasets are clean, or with a negligible number of noisy labeled data instances [22]. To evaluate the effectiveness of NDCC in noisy labeled environments, we consider two types of noise: (i) **Symmetric Noise**: the true label flips to all other labels with equal probability. The symmetric noise simulates the label noise caused by a random mistake in the labeling process; (ii) **Asymmetric Noise**: a noisy label is generated by flipping the true label to the next class (i.e., label $i \leftarrow i + 1; mod\ K$) [21]. In both cases, τ denotes the noise rate. We consider $\tau \in \{0.2, 0.4, 0.6, 0.8\}$ to evaluate NDCC on scenarios involving a variable number of noisy labeled data instances (ranging from small to large).

3) Experimental Setup: All experiments use ResNet34 and the following hyper–parameter values: mini–batch size (32), number of training epochs (90), optimizer (AdamW [45]), learning rate (0.01). In NDCC, we set $T = 3$ and $T_n = 30$ in Algorithm 3 to ensure that the overall number of training epochs for NDCC is the same as with the baselines (i.e., 90). The counterfactual training epoch T_{cf} in Algorithm 2 is set to 50. In all experiments involving CIFAR–10, CIFAR–100, and Fashion–MNIST datasets, we randomly select $2,000$ data instances as the clean pre–trained dataset D_{pre} to train g using the following hyper–parameters: mini–batch size (32), number of training epochs (50), and optimizer (AdamW). Among the remaining data instances, we randomly select $10,000$ as the training set D. We use the default test set for CIFAR–10, CIFAR–100, and Fashion–MNIST. For CIFAR–100, we focus on the 20 superclasses. For Clothing1M, we randomly select $2,000$ clean instances as D_{pre}, and $10,000$ as $\tilde{D}$ (i.e., $5,000$ data instances are randomly selected from the clean training set, and $5,000$ are randomly selected from the noisy labeled set). We experiment with subsets of CIFAR–100 and Clothing1M to demonstrate NDCC's effectiveness in real–world noisy datasets. However, NDCC cannot readily handle the complete number of data instances in these datasets because of its computational complexity (Section V). Naively parallelizing the generation of counterfactual labels for each data instance may improve NDCC's scalability, however this is out of scope of this work.

4) Baselines: **CE (Cross Entropy)** uses cross entropy loss, and has no particular strategy for handling noisy labeled data instances. **CE–Clean** uses solely clean data instances for training, and thus achieves the theoretical best performance. **CORES (Confidence Regularized Sample Sieve) [21]** uses peer loss to detect suspected noisy labeled data instances without unsupervised training. **AUM (Area Under the Margin) [22]** uses the AUM statistic to exploit the differences between the clean and noisy labeled data instances. AUM excludes the detected noisy data instances from the training process. **NN–Correction (Nearest neighbor noisy label correction) [46]** uses the same noisy detection module as NDCC, but noise label correction is performed using k–nearest neighbors. **Loss Correction Procedures** that are both application domain and network architecture agnostic were introduced in [47] for loss correction. Both procedures require a priori knowledge, or estimation, of a stochastic matrix T that summarizes the probability of one class being mislabeled as another under noise. The first procedure, **Backward** $\hat{T}$, multiplies the loss by T^{-1}, whereas the second procedure, **Forward** $\hat{T}$, multiplies network predictions by T. In both cases, an estimate of matrix T, denoted $\hat{T}$, is obtained and used instead of T [47].

5) Evaluation Metrics: We divide the evaluation process into three parts: (i) noise detection, (ii) noise correction, and (iii) overall accuracy on the clean test set under different types of label noise in the training set.

Recall X_n denotes the accumulated detected noisy data set with respect to all learning rounds T_{all}, and $\tilde{D}$ is the noisy input data set. Let $X_{\tilde{D}}$ denote the true noisy data set. We introduce the following score to evaluate noise detection performance: (i) **True detection rate**: $X_{dt} = \frac{|X_n \cap X_{\tilde{D}}|}{|X_{\tilde{D}}|}$ measures the ratio of truly identified noise data instances; (ii) **Wrong detection rate**: $X_{dw} = \frac{|X_n \cap (\tilde{D} - X_{\tilde{D}})|}{|X_n|}$ measures the ratio of misidentified clean data instances as noisy; (iii) **Miss detection rate**: $X_{dm} = \frac{(|\tilde{D} - X_n) \cap X_{\tilde{D}}|}{|X_{\tilde{D}}|}$ measures the ratio of noisy data instances identified as clean. In Section VI-B, we only discuss X_{dt} and X_{dw}, since X_{dm} can be directly derived from X_{dt} by $X_{dm} = 1 - X_{dt}$.

In noise correction, we check whether NDCC can correctly assign the true labels to corresponding detected noisy data

TABLE II
TRUE DETECTION RATE X_{dt} (HIGHER IS BETTER)

approach/NS Environment (τ)	**Fashion–MNIST**								**CIFAR–10**								**CIFAR-100**
	Sym				**Asym**				**Sym**				**Asym**				**sym**
	0.2	**0.4**	**0.6**	**0.8**	**0.2**	**0.4**	**0.6**	**0.8**	**0.2**	**0.4**	**0.6**	**0.8**	**0.2**	**0.4**	**0.6**	**0.8**	**0.6**
CORES	0.56	0.55	0.57	0.59	0.51	0.50	0.56	0.57	0.60	0.58	0.61	0.59	0.64	0.61	0.62	0.57	0.58
NDCC	**0.76**	**0.75**	**0.75**	**0.77**	**0.67**	**0.68**	**0.72**	**0.74**	**0.80**	**0.81**	**0.81**	**0.84**	**0.78**	**0.76**	**0.78**	**0.82**	**0.65**

TABLE III
WRONG DETECTION RATE X_{wt} (LOWER IS BETTER)

approach/NS Environment	**Fashion–MNIST**								**CIFAR–10**								**CIFAR–100**
	Sym				**Asym**				**Sym**				**Asym**				**Sym**
	0.2	**0.4**	**0.6**	**0.8**	**0.2**	**0.4**	**0.6**	**0.8**	**0.2**	**0.4**	**0.6**	**0.8**	**0.2**	**0.4**	**0.6**	**0.8**	**0.6**
CORES	**0.13**	**0.07**	**0.02**	**0.01**	**0.13**	**0.09**	**0.05**	**0.03**	**0.18**	**0.10**	**0.09**	0.07	**0.20**	**0.14**	**0.09**	**0.03**	**0.26**
NDCC	0.17	0.15	0.07	0.04	0.16	0.14	0.13	0.09	0.26	0.17	0.14	**0.04**	0.27	0.21	0.17	0.09	0.30

TABLE IV
COUNTERFACTUAL TRUE CORRECTION RATE $\hat{X}_{cf_c}$ (HIGHER IS BETTER)

approach/NS Environment	**Fashion–MNIST**								**CIFAR–10**								**CIFAR–100**
	Sym				**Asym**				**Sym**				**Asym**				**Sym**
	0.2	**0.4**	**0.6**	**0.8**	**0.2**	**0.4**	**0.6**	**0.8**	**0.2**	**0.4**	**0.6**	**0.8**	**0.2**	**0.4**	**0.6**	**0.8**	**0.6**
NN-correction	0.24	0.11	0.04	0.01	0.19	0.12	0.05	0.02	0.28	0.13	0.04	0.04	0.22	0.09	0.04	0.01	0.02
NDCC	**0.62**	**0.61**	**0.59**	**0.60**	**0.57**	**0.55**	**0.54**	**0.56**	**0.68**	**0.70**	**0.72**	**0.71**	**0.67**	**0.65**	**0.68**	**0.70**	**0.47**

TABLE V
DECREASED NOISY RATE d_τ (LOWER IS BETTER)

approach/NS Environment	**Fashion–MNIST**								**CIFAR–10**								**CIFAR–100**
	Sym				**Asym**				**Sym**				**Asym**				**Sym**
	0.2	**0.4**	**0.6**	**0.8**	**0.2**	**0.4**	**0.6**	**0.8**	**0.2**	**0.4**	**0.6**	**0.8**	**0.2**	**0.4**	**0.6**	**0.8**	**0.6**
CORES	0.07	0.16	0.22	0.23	0.06	0.14	0.19	0.23	**0.08**	**0.17**	0.22	0.24	**0.09**	**0.18**	0.21	0.23	0.20
NN-correction	0.02	0.01	-0.03	-0.02	0.01	0.01	-0.03	-0.02	0.01	0.03	-0.03	-0.01	0.02	-0.02	-0.02	-0.02	-0.04
NDCC	**0.08**	**0.21**	**0.30**	**0.38**	**0.06**	**0.18**	**0.27**	**0.35**	0.06	**0.17**	**0.29**	**0.44**	0.07	0.16	**0.26**	**0.42**	**0.23**

instances. Let $\hat{X}_r$ denote the data set where NDCC correctly pair detected noisy data instances with their true labels, and $\hat{X}_w$ denote the detected noisy data instances that are assigned wrong labels. We define the following two scores: (i) **True counterfactual label correction rate** $\hat{X}_{cf_c} = \frac{|\hat{X}_r|}{|X_n|}$, and (ii) **False counterfactual label correction rate** $\hat{X}_{cf_w} = \frac{|\hat{X}_w|}{|X_n|}$.

Finally, we measure the **decreased noisy labeled rate** d_τ after applying the noisy label detection of baselines and NDCC, and test the accuracy of each trained learning model f. d_τ for CORES is computed as $d_\tau = \tau - \frac{|X_{\tilde{D}}|X_{dm}}{|\tilde{D}|-|X_n|}$, where $|\tilde{D}| - |X_n|$ denotes the number of currently available training data instance, excluding the detected noisy data instances, and $|X_n|X_{wd} + |X_{\tilde{D}}|X_{dm}$ denotes the remaining miss detected noisy data instances. d_τ for NDCC and NN–Correction is defined as:$d_\tau = \tau - \frac{|X_{\tilde{D}}|X_{dm}+|X_n|\hat{X}_{cf_w}}{|\tilde{D}|}$, where $|X_n|\hat{X}_{cf_w}$ is seen as noisy data instance because of correction failure.

We additionally perform a Wilcoxon signed–rank test [48] to compare NDCC with the baselines across datasets. Specifically, let $\mathtt{d}_i$ denote the score difference between two classifiers in the $i-$th dataset ($1 \leq i \leq N$). Difference scores are ranked by absolute value, and the average rank is assigned if two classifiers are tied. The sum of ranks for the datasets where the second classifier outperforms the first is denoted as $\mathtt{R}^+$. Conversely, the sum of ranks for the datasets where the first classifier outperforms the second is denoted as $\mathtt{R}^-$. When $\mathtt{d}_i = 0$, the numbers are assigned evenly to $\mathtt{R}^+$ and $\mathtt{R}^-$. Thus $\mathtt{R}^+ = \sum_{d_i>0} rank(d_i) + \frac{1}{2}\sum_{d_i=0} rank(d_i)$, and $\mathtt{R}^- = \sum_{d_i<0} rank(d_i) + \frac{1}{2}\sum_{d_i=0} rank(d_i)$. The table of critical values for $R_{crtc} = \min(\mathtt{R}^+, \mathtt{R}^-)$ [49] is then used to determine whether to reject the hypothesis that the first classifier is better than the second.

Finally, we perform a Friedman test to quantitatively evaluate the statistical significance of classification performance achieved by the various methods. Specifically, given an evaluation metric, the rank of each classifier is computed. Let R_{C_j} denote the average rank of classifier j. The chi–square value is then computed as $\chi_F^2 = \frac{12N_C}{k_C(k_C+1)}[\sum_{j=1}^{k_C} R_{C_j}^2 - \frac{k_C(k_C+1)^2}{4}]$, where N_C is the number of datasets, and k_C is the total number of classifiers. The Friedman score is computed as $F_S = \frac{(N_C-1)\chi_F^2}{N_C(k_C-1)-\chi_F^2}$. If F_S is greater than the critical value given by the chi–square critical value table [50], the hypothesis that "the classifiers have the same performance on the datasets" can be rejected. In the event that the Friedman test concludes that "the classifiers are statistically different", we perform a Nemenyi test [51] to find out which classifiers are statistically different. Specifically, we compute the difference between the average ranks for any pair of two classifiers. If the difference is greater than or equal to the critical difference (CD) value, we conclude that the two classifiers are significantly different from each other. The CD value is calculated as $CD = q_\alpha\sqrt{\frac{k_C(k_C+1)}{6N_C}}$, where q_α is the critical value given by the look–up table.

B. Experimental Results

1) Influence of Size of D_{pre}: We begin by exploring the influence of the size of data instances in D_{pre}. The main impact of training g with different sizes of D_{pre} is on the selection of the label threshold, which is used to detect possible noisy labeled data instances. As shown in Figure 9, the selected label threshold becomes more robust, and the true detection rate increases (similarly, wrong detection rate decreases) with increasing size of D_{pre}. However, since obtaining D_{pre} may be time consuming and/or costly, keeping

TABLE VI
EXPERIMENTS RESULT OF TEST ACCURACY.

approach/NS Environment	Fashion–MNIST								CIFAR–10								CIFAR–100
	Sym				Asym				Sym				Asym				Sym
	0.2	**0.4**	**0.6**	**0.8**	**0.2**	**0.4**	**0.6**	**0.8**	**0.2**	**0.4**	**0.6**	**0.8**	**0.2**	**0.4**	**0.6**	**0.8**	**0.6**
CE	0.63	0.49	0.28	0.11	0.58	0.42	0.27	0.12	0.52	0.41	0.26	0.12	0.59	0.43	0.27	0.12	0.14
CORES	0.71	0.65	0.56	0.39	0.75	0.69	0.64	0.48	0.67	0.62	0.52	0.41	0.69	**0.65**	0.56	0.42	0.41
AUM	**0.75**	**0.69**	0.58	0.35	**0.79**	**0.71**	0.63	0.41	**0.68**	**0.63**	0.55	0.37	**0.71**	**0.65**	0.57	0.36	**0.48**
NN-Correction	0.63	0.45	0.22	0.10	0.60	0.43	0.25	0.12	0.54	0.40	0.19	0.09	0.60	0.37	0.21	0.09	0.06
NDCC	0.72	0.65	**0.59**	**0.52**	0.75	0.70	**0.65**	**0.54**	0.65	0.60	**0.56**	**0.49**	0.67	0.63	**0.58**	**0.51**	0.45
CE-Clean	0.78	0.76	0.69	0.64	0.81	0.77	0.72	0.65	0.70	0.66	0.60	0.55	0.73	0.69	0.64	0.57	0.59

TABLE VII
ACCURACY COMPARISON WITH BASELINES BACKWARD $\hat{T}$ AND FORWARD $\hat{T}$ FOR $|\tilde{D}| = 10,000$ ON CIFAR–10 AND CIFAR–100.

approach/NS Environment	CIFAR–10				CIFAR–100
	Sym		Asym		Sym
	0.2	**0.6**	**0.2**	**0.6**	**0.6**
Backward $\hat{T}$	0.67	0.51	0.67	0.50	0.38
Forward $\hat{T}$	**0.69**	**0.57**	**0.72**	0.59	0.34
NDCC	0.65	0.56	0.67	**0.68**	**0.45**

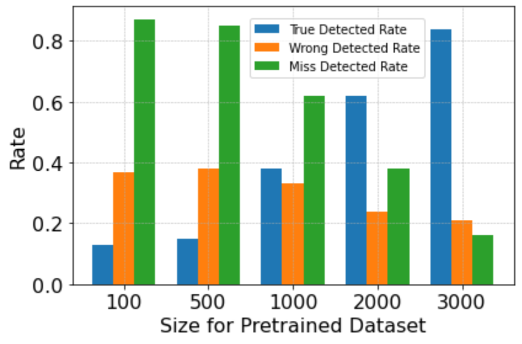


Fig. 9. NDCC's true detection rate (X_{dt}), wrong detection rate (X_{dw}), and miss detection rate (X_{dm}) for the first epoch on CIFAR–10 with symmetric noise $\tau = 0.2$.

its size small is important. The main factor in determining the minimum size of D_{pre} is that X_{dt} should be at least twice the value of X_{wt} when the lowest $\hat{X}cf_c$ is 0.47 (the lowest value in our experiments). Otherwise, more clean data instances will be treated as noisy labeled. In this context, our experiments are performed with $|D_{pre}| = 2,000$.

TABLE VIII
EXPERIMENT RESULTS ON CLOTHING1M OF X_{dt}, X_{dw}, X_{dm} AND $\hat{X}_{cf_c}$ IN FIRST RUNNING EPOCH.

Dataset	X_{dt}	X_{dw}	X_{dm}	$\hat{X}_{cf_c}$
Clothing1M	0.52	0.19	0.48	0.47

2) Noisy Label Detection: We next compare NDCC and CORES. Table II and III show the true and wrong detection rates, X_{dt} and X_{dw}, for CORES and NDCC. A larger value of X_{dt} and smaller value of X_{dw} indicate better performance, as the goal is to detect as many true noisy labeled data instances as possible, while keeping the number of wrong detections low. Compared with CORES, NDCC's true detection rate X_{dt} increases almost three times more than X_{dw}, illustrating that automatically selecting the loss threshold is beneficial, as opposed to using a fixed threshold, as in [21].

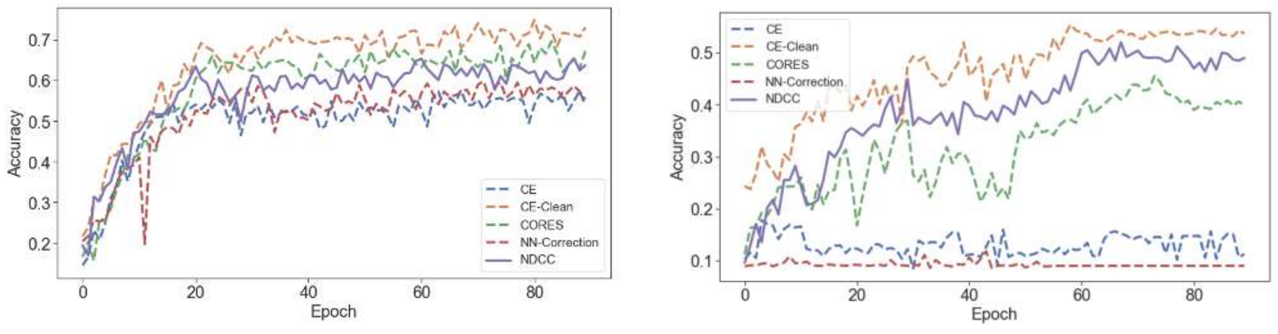
Performing a Wilcoxon test with respect to true detection rate (i.e., using Table II), we have $R^+ = 0$, $R^- = 153$, resulting in $R_{crtc} = 0$. The critical value for the confidence level of 99.95 given by the look–up table is 51. Since $R_{crtc} < 51$, we can conclude that NDCC is better than CORES in terms of true detection rate. Similarly, a Wilcoxon test with respect to wrong detection rate (i.e., using Table III) gives $R^+ = 150$, $R^- = 3$, resulting in $R_{crtc} = 0$. Since $R_{crtc} < 51$, CORES is better than NDCC with respect to wrong detection rate.

3) Noisy Label Correction: We next measure the effectiveness of NDCC's counterfactual label correction module by comparing the label correction results between NN–Correction and NDCC. Table IV shows that, for both Fashion–MNIST and CIFAR–10, NDCC's $\hat{X}_{cf_c}$ is much higher than NN–Correction, especially as τ increases. The performance of NN–Correction is unsatisfactory because clusters become unreliable in the presence of noisy labeled data instances. Instead, NDCC's superiority is confirmed with a stable $\hat{X}_{cf_c}$ score, even in severe noisy environments (i.e., $\tau = 0.6, 0.8$). Finally, Table V shows the decreased noisy rate that different approaches achieve. NDCC outperforms all baselines in all noisy environments across both datasets.

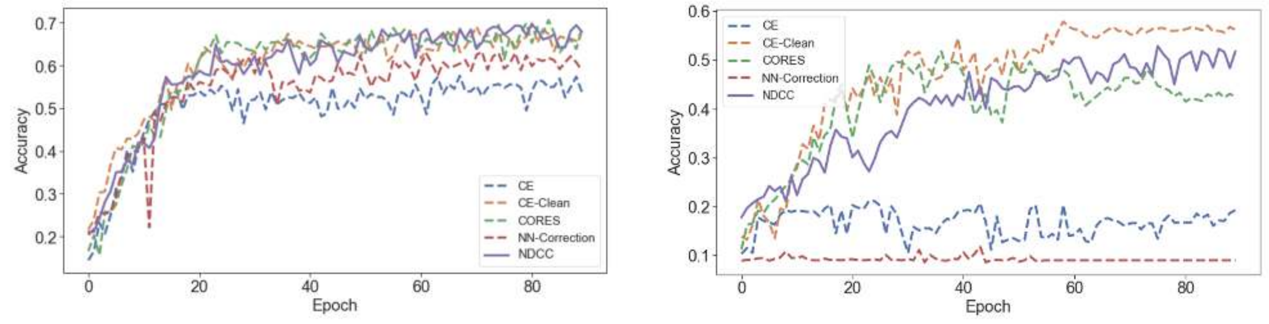
Performing a Wilcoxon test with respect to counterfactual true correction rate (i.e., using Table IV), we get $R_{crtc} = 0$, since $R^+ = 0$, $R^- = 153$. Since R_{crtc} is less than the critical value of 51 for confidence level of 99.95, we conclude that NDCC is better than NN–Correction in terms of counterfactual true correction rate.

Finally, we perform a Friedman test for the three classifiers using Table V. The calculated Friedman value is 28.72. Compared with the value of 9.294 in the look–up table for 99% confidence, we observe that the three classifiers perform unequally on the datasets. Therefore, we perform a Nemenyi test to compare their performance using the average rank of each classifier. The critical value for the Nemenyi test is 0.80. The calculated Nemenyi value indicates the classifiers that are statistically different are as follows: (CORES, NN–correction); (NN–correction, NDCC).

4) Overall Evaluation: Table VI shows the accuracy of NDCC and the baselines. CE, which does not at all perform noisy detection, is expected to be the least performing approach. CE–Clean intentionally uses only clean data instances for training, and is therefore expected to perform ideally. For both Fashion–MNIST and CIFAR–10, NDCC outperforms the baselines when noise becomes severe (i.e., $\tau \geq 0.6$) in both asymmetric and asymmetric case. Figures 10(b) and 10(d) in particular, show that NDCC achieves close to the best performance, which would only be achievable if all training data instances were clean. In light noisy environments (i.e., $\tau \leq 0.4$), the performance of NDCC is close to AUM, the

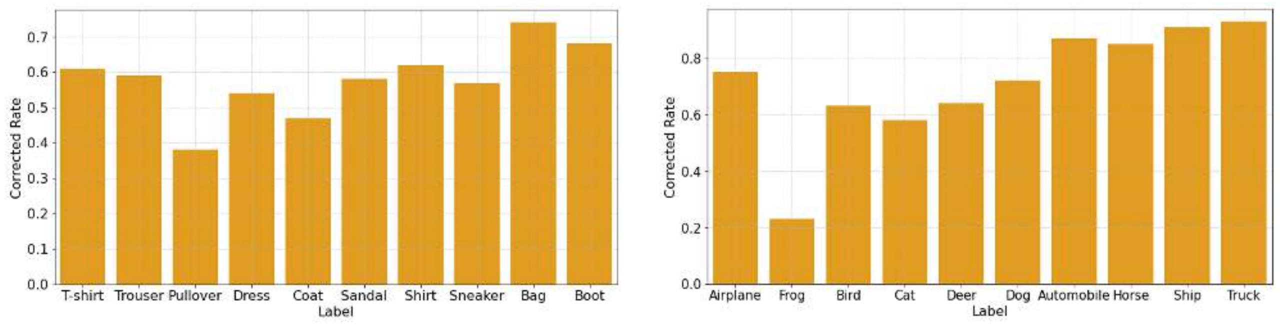


(a) Symmetric noise, NR = 0.2 (b) Symmetric noise, NR = 0.8



(c) Asymmetric noise, NR = 0.2 (d) Asymmetric noise, NR = 0.8

Fig. 10. Test accuracy plots with increasing learning epochs in CIFAR–10 dataset with noise rate (NR) equals 0.2 (left column) and 0.8 (right column).



(a) Fashion–MNIST, Symmetric noise, NR = 0.8 (b) CIFAR–10, Symmetric noise, NR = 0.8

Fig. 11. True corrected rate per label plots in Fashion–MNIST and CIFAR–10 datasets.

best performing baseline. Figures 10(a) and 10(c) show that both NDCC and CORES perform similarly to CE–Clean. The reason is that clean data instances comprise a large portion of training data instances under small noisy rate environment. However, compared with NN–Correction and CE, the accuracy of NDCC is higher, illustrating the effect of neither dealing with noisy labeled instances at all (i.e., CE) as well as using a naive label correction approach (i.e., NN–Correction). As for CIFAR–100, AUM is the best performing method, with NDCC being a close second. Figure 11 shows the final correction rate per label of CIFAR–10 and Fashion–MNIST. In CIFAR–10, Pullover is the most difficult class to learn, whereas the highest label correction rate is achieved for class Bag. In Fashion–MNIST, Frog has the lowest label correction rate, as opposed to transportation related classes. Table VII shows the comparison results of NDCC with the loss correction procedures described in [47]. Forward $\hat{T}$ performs better when the noise rate is low (e.g., $\tau = 0.2$). However, in a highly noisy environment (e.g., $\tau \geq 0.6$), NDCC performs similarly (i.e., when noise is symmetric), or noticeably better (under an assymetric noise environment) than Forward $\hat{T}$. We attribute the performance degradation of Backward and Forward baselines to the quality of estimated matrix T (i.e., $\hat{T}$). This is due to the training of the network directly on noisy labeled dataset. When the noise rate is high, more labels are noisy and thus a biased network is trained, leading to a flawed estimation of matrix T (i.e., $\hat{T}$).

A Friedman test using Table VI results in a Friedman value of 74.96. The Friedman value for 99% confidence given by the look–up table is 15.086, which is smaller than the calculated value, meaning the three classifiers perform unequally on the datasets. Therefore, we perform a Nemenyi test to compare their performance. The average ranks for CE, CORES, AUM, NN-Correction, NDCC, and CE–Clean are 5.23, 3.29, 2.76, 5.64, 2.76, and 1, respectively. The critical value given for the Nemenyi test is 1.82. The calculated Nemenyi value indicates the classifiers that are statistically different are as follows: (CE, CORES); (CE, AUM); (CE, NDCC); (CE, CE–Clean); (CORES, NN–Correction); (CORES, CE–Clean); (AUM, NN–Correction); (NN–Correction, NDCC), (NN–Correction, CE–Clearn). In terms of average rank, CE–Clean achieves the highest rank on every dataset, as well as overall (i.e., average rank) since it is the theoretical the upper bound. In addition, NDCC attains a higher average rank than CE, AUM, CORES, and NN–Correction. Table VIII shows the experiment result for real–world noisy dataset (i.e., Clothing1M), which illustrates the efficiency for NDCC when dealing with real–world noisy. In summary, the experimental results confirm that NDCC can both effectively detect (and correct) noisy labeled data instances, and train robust classifiers even in the presence of severe label noise in the training set.

VII. Conclusion

We presented a new approach for robust learning in the presence of noisy labeling data. Specifically, we proposed an automatic noisy peer loss threshold selection approach to separate noisy labeled data instances from clean data instances. We additionally proposed to leverage counterfactual learning to correct detected noisy labeled data instances by pairing them with their most likely true labels. Our experimental results show the superiority of the proposed approach over the baselines, particularly in severe label noise environments.

In future work, we wish to reduce our approach's dependency on a model pre–trained with clean training data. Even though this is a commonly adopted strategy in noisy learning, we believe that eliminating the need for manual annotation and human inspection can benefit noisy learning by allowing models to be trained on less circumscribed domains (e.g., car financing) that are much "messier" than domains with clear ground truth (e.g., computer vision or natural language processing). We additionally wish to evaluate the scalability of our proposed approach using larger and more diverse datasets.

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APPENDIX

We show that the optimal solutions for $\mathbf{W}$ and $\hat{\mathbf{x}}$ can be acquired in an iterative manner by alternating search, according to Equation 1 and the following lemma.

Lemma 1: The optimal solution found by minimizing $\mathbf{W}$ first and then minimizing $\hat{\mathbf{x}}_\mathbf{i}$ is the same as the optimal solution found by jointly minimizing $\mathbf{W}$, $\hat{\mathbf{x}}_\mathbf{i}$.
The parameters $\mathbf{W}$ of the learning model and the optimal counterfactual data instance $\hat{\mathbf{x}}_\mathbf{i}$ with respect to x_i are independent. The global optimum $\hat{\mathbf{x}}_\mathbf{i}$ is unknown and pre–existed, and we wish to use learning model with parameters $\mathbf{W}$ to find it. Therefore, the simultaneously obtained optimal solutions are $\mathbf{W}^*$ and $\hat{\mathbf{x}}_i^*$. Global minimum $g(\mathbf{W}^*, \hat{\mathbf{x}}_\mathbf{i}^*)$ satisfies the following equation:

$$g(\mathbf{W}^*, \hat{\mathbf{x}}_\mathbf{i}^*) \leq \min_{\hat{\mathbf{x}}_\mathbf{i}} \min_{\mathbf{W}} g(\mathbf{W}, \hat{\mathbf{x}}_\mathbf{i}). \tag{5}$$

Because for any $\hat{\mathbf{x}}_i$, $\min_{\hat{\mathbf{x}}_i} g(\mathbf{W}, \hat{\mathbf{x}}_i) \leq g(\mathbf{W}^*, \hat{\mathbf{x}}_i)$,

$$\min_{\hat{\mathbf{x}}_\mathbf{i}} \min_{\mathbf{W}} g(\mathbf{W}, \hat{\mathbf{x}}_\mathbf{i}) \leq \min_{\hat{\mathbf{x}}_\mathbf{i}} g(\mathbf{W}^*, \hat{\mathbf{x}}_\mathbf{i}) \tag{6}$$

and

$$\min_{\hat{\mathbf{x}}_\mathbf{i}} g(\mathbf{W}^*, \hat{\mathbf{x}}_\mathbf{i}) \leq g(\mathbf{W}^*, \hat{\mathbf{x}}_\mathbf{i}^*). \tag{7}$$

Combining Equations (6) and (7), we get

$$\min_{\hat{\mathbf{x}}_\mathbf{i}} \min_{\mathbf{W}} g(\mathbf{W}, \hat{\mathbf{x}}_\mathbf{i}) \leq g(\mathbf{W}^*, \hat{\mathbf{x}}_\mathbf{i}^*). \tag{8}$$

Finally, comparing Equations (5) and (8), we get

$$g(\mathbf{W}^*, \hat{\mathbf{x}}_\mathbf{i}^*) = \min_{\hat{\mathbf{x}}_\mathbf{i}} \min_{\mathbf{W}} g(\mathbf{W}, \hat{\mathbf{x}}_\mathbf{i}) \tag{9}$$

Using Lemma 1, instead of showing $g(\mathbf{W}, \hat{\mathbf{x}}_\mathbf{i})$ is convex for $\mathbf{W}$ and $\hat{\mathbf{x}}_\mathbf{i}$ simultaneously, we can show that $g(\mathbf{W}, \hat{\mathbf{x}}_\mathbf{i})$ is convex with respect to $\mathbf{W}$ and $\hat{\mathbf{x}}_\mathbf{i}$ separately.